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Mathematical Formulation for the Estimation of Volumetric Heat Transfer in Precision Machining Operations

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Abstract: *The objective of this work is to estimate the volumetric heat diffusivity and volumetric heat transfer at localised point, during precision machining. This enables the calculation of heat transmission at different points on work-piece during machining operations. Volumetric heat transfer is computed by assuming a infinitesimally small point of interaction between the work-piece and the precision tool. The heat transfer in materials causes vibrations therein, resulting in the expansion of atomic bonds and thereby the changed dimensions at the atomic levels. Thus the the heat flux generated during the machining causes in the expansion of work-piece. This expansion is volumetric at molecular levels (linear and lateral dimensions). Thus, the estimation of the volumetric heat transfer enables in the computation of volumetric expansion of materials at localised domains of thermal energy generation.*

1. INTRODUCTIONSINGLE POINT DIAMOND TURNING

High precision deterministic finishing methods are of utmost importance and are the need of present manufacturing scenario. The need for high precision in manufacturing is felt by manufacturers worldwide to improve interchangeability of components, improve quality control and longer wear and fatigue life. Taniguchi reviewed the historical progress of achievable machining accuracy during the last century.

Single Point Diamond Turning is a complex process which involves relative motion between tool and work-piece. A significant amount of power is consumed in overcoming the ultimate shear strength of work-piece. This deformation at the shear plane takes at high strain rates and generates a large amount of heat. Though heat is produced throughout the machined surface radially, the thermal flux is generally observed and analysed at three machining zones: primary, secondary and tertiary zones.

Most of the analytical studies use Bolk's analytical model for the prediction of temperature and heat distribution on metal cutting process. Some significant percentage of heat flux produced enters the work-piece. This can have critical impact on dimensional accuracy and on the surface quality of the work-piece. Residual heat raises the need for over-tolerant specifications on the parts or requires post-processing in order to remove residual thermal stress. It may reduce the fatigue life of the machine components as a result of residual stress.

It is vital to find a fast and suitable solution to predict residual heat in a machined component, given the complexity of the interplay of the process parameters and material properties. Analytical modelling of residual heat penetrating into the work-piece may be a solution to this critical problem. This may compensate to some extent, the issues of material swelling and recovery as they arise as a result of residual heat contained in work-piece after completion of machining cycle. Analytical methods are quite useful in understanding the machining process. Most of the analytical models are based on a series of simplifying assumptions. Without these assumptions, the resulting mathematical equations to describe the process become complicated. So, a steady state thermal model, which shows the temperature distribution along the axis of work-piece and rate of heat along the axis of work-piece that contributes to the residual stress, are presented in this work.

The focus of this work is to find the effect of residual thermal fluxes, which enters the volume of work-piece.

To find the volumetric heat transmission through a single point, one should derive the volumetric heat diffusivity.

2. DERIVATION FOR CALCULATION OF VOLUMETRIC DIFFUSIVITY

Volumetric heat diffusivity can be defined as the transfer of heat through unit volume per second due to temperature gradient. Its unit is mm^3s^{-1}.

To calculate the volumetric heat transmission, let us assume a single element, that shows the depth of cut and feed direction (Fig.1).

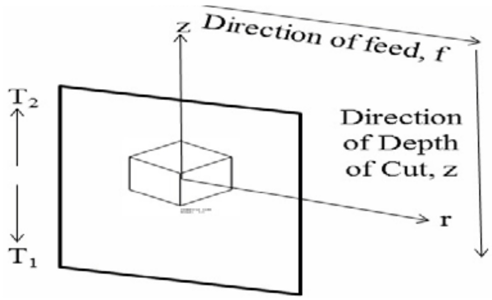

Fig. 1: Direction of depth of cut and feed

T_1 and T_2 = The resultant temperatures of work-piece and tool respectively after considering the thermal effusivity between tool and work-piece.

In Thermodynamics, the thermal effusivity of a material is defined as the square root of the product of the material's thermal conductivity and its volumetric heat capacity.

$$e = \sqrt{\boldsymbol{k\rho C_p}}$$

Where:

e = Thermal Effusivity

k = Thermal Conductivity

ρC_p = Volumetric Heat Capacity

Thus, the temperature at the contact between tool and work-piece is given by,

$$T \text{ or } T_m = T_1 + (T_2 - T_1)\frac{\boldsymbol{e_2}}{(\boldsymbol{e_2 - e_1})}$$

Thus the temperature T_1, alone is accounted as the parameter to calculate heat transfer.

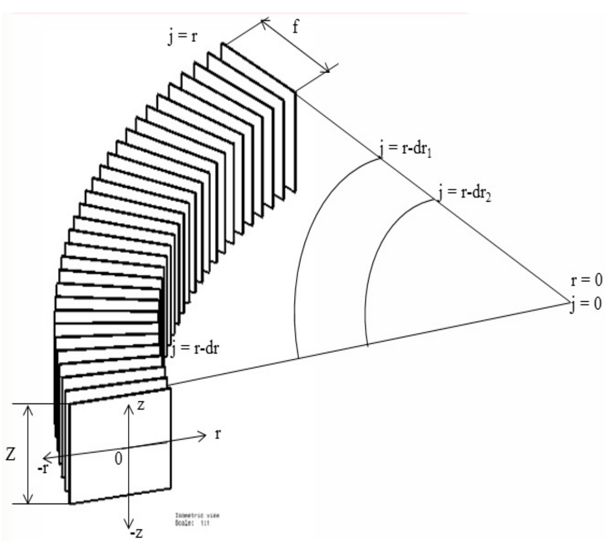

Fig. 2: Elemental feed and depth of cut

The feed and depth of cut for 'n' number of feeds and depth of cut (Fig.2). By considering 'n' elements, we can conclude that,

- The *feed* is along 'r' direction *(inward)*
- The *depth of cut* is along 'z' direction. *(Downwards)*

The depth of cut at different values of 'i' is shown in the Fig.3.

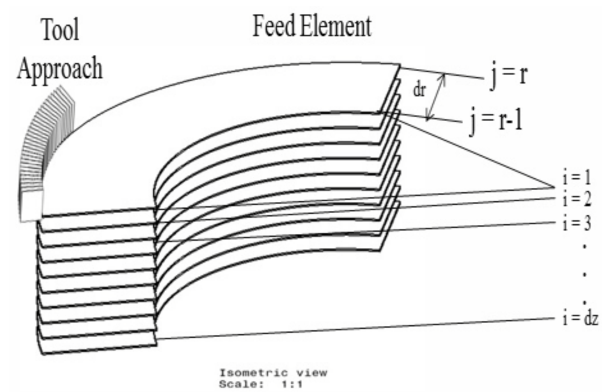

Fig. 3: Elemental depth of cut planes

From this, the following inference are made:

At various values of i and j, the heat generation varies, due to larger interaction at outermost circumference, and progressively reducing initeraction (with the reducing radius of work-piece), while the tool moves towards the cetre of the work-piece.

By considering the above scenario, the volumetric diffusivity *(D_V)* at given diffusivity and conductivity and for given heat transfer and temperature gradient is given as:

$$\boldsymbol{D_V} = -\frac{\boldsymbol{Q}}{\boldsymbol{dT}} \times \frac{\boldsymbol{\alpha}}{\boldsymbol{k}}$$

Where,

Q = Heat Transfer

dT = Temperature Gradient

α = Thermal Diffusivity

k = Thermal Conductivity

The heat transfer relation is given as:

$$\boldsymbol{Q = -k \times A \times \frac{dT}{dx}}$$

From which we can deduce $\frac{Q}{dT}$ as:

$$\frac{\boldsymbol{Q}}{\boldsymbol{dT}} = \frac{\boldsymbol{-k \times A}}{\boldsymbol{dx}}$$

$$D_V = -\left(\frac{-k\times A}{dx}\times \frac{\alpha}{k}\right)$$

$$D_V = \frac{\alpha\times A}{dx}\, m^3 s^{-1}$$

Here, dx = dr, since the feed changes towards the radius of the work-piece. A varies with respect to radius r.

Thus, the volumetric d iffusivity becomes:

$$D_V = \frac{\alpha\times dA}{dr}\ m^3s^{-1}, \text{ for all } (0 < r < r-1)$$

The above equation is not applicable for (i=1, 2, 3, …., n, dz and j = r), as there is no change in radius at initial state, (i.e., dr = 0), causes D_V to be unpredictable, (i.e., $D_V = \infty$).

Since, area (A) varies due to change in radius towards center point of the work-piece.

$$=> dA = \pi dr^2$$

$$\frac{dA}{dr} = \pi dr$$

By substituting the value of $\left(\frac{dA}{dr}\right)$ in the Eqn.for D_V, we get,

$$D_V = \pi.\alpha.dr\ m^3s^{-1}, \text{ for all } (0 < r < r-1)$$

Where,

D_V = volumetric diffusivity through the material.

α = thermal diffusivity of the material.

dr = change in radius towards center of the work-piece.

Since the radius is the function of feed, the change in radius dr can be written as feed f for each values of j's, (j=1, 2, 3, ….. r).

For every radius irrespective of the depth of cut and speed, the volumetric diffusivityis defined by feed only. Thus D_V is given by,

$$D_V = \pi.\alpha.f\ m^3s^{-1} \text{ @each values of 'j'}$$

Thus 'D_V' is directly proportional to the change in radius dr, at each values of j, for consecutive cycle of operation, the heat generated before j-1th cycle is accounted. Thus total volumetric diffusivity is given by:

$$D_V = \pi.\alpha.f\times\frac{r}{f}$$

The term $\frac{r}{f}$ gives the value of number of feed movement through the radius r. Thus we have:

$$D_V = \pi.\alpha.r$$

3. DERIVATION FOR CALCULATION FOR VOLUMETRIC HEAT TRANSMISSION

Applying heat transfer equation in general form as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\frac{\partial T}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\theta_{i,j}}{k_z} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The generated thermal flux is transmitted along z-axis. Thus, assuming no radial thermal component, the above equation reduces to:

$$\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\theta_{i,j}}{k_z} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Since, the diffusivity is assumed to be volumetric at molecular levels, the term 'α' becomes 'D_V'. Thus the above Equn. becomes:

$$\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\theta_{i,j}}{k_z} = \frac{1}{D_V}\frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\theta_{i,j}}{k_z} = \frac{1}{\pi.\alpha.dr}\frac{\partial T}{\partial t} \qquad Km^{-3}$$

The term $\frac{1}{\pi.\alpha.dr}\frac{\partial T}{\partial t}$ provides the value of volumetric heat transmission through the material. In order to equalize units of parameters, the L.H.S is divided by term $\pi.dr$. Thus, the above equation becomes:

$$\frac{1}{\pi.dr}\left[\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\theta_{i,j}}{k_z}\right] = \frac{1}{\pi.\alpha.dr}\frac{\partial T}{\partial t}$$

We know that, $\alpha = \frac{k}{\rho\times C_p}$. Thus the above Equn. can be written as:

$$\frac{1}{\pi.dr}\left[\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)+\frac{\theta_{i,j}}{k_z}\right] = \frac{\rho C_p}{\pi.k.dr}\frac{\partial T}{\partial t}$$

Where,

ρ = Density of the material,

C_p = Specific heat capacity at constant pressure,

K = Thermal conductivity of the material.

4. CONCLUSION

The calculation of volumetric heat transfer at the pin point contact of tool and work-piece at the molecular level help in finding the heat affected zone in the work-piece.

In future work, the calculation of material swelling due to heat transfer will be computed. The future plans also include, compensation of the swelling by varying the volumetric swelling rate to obtain the nano-level form finish of the resultant surface during precision machining.

5. ACKNOWLEDGEMENT

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